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Six Sigma Green Belt Study Guide Exam CSSGB

15 Six Sigma Analyze Phase

Describe the analyze phase

Data collected during the Measure phase must be analyzed using statistical tools to discover causes of problems and to find solutions for them. In this topic, you will describe the Analyze Phase and the tools and deliverables of this phase.

To identify causes of problems in a process and to come up with solutions that address them, you should analyze and interpret data collected in the Measure phase. Understanding the tools that are used for this purpose will help you apply the right tool and analyze root causes of quality problems.

What are the goals of analyze phase?

The goal of the Analyze phase is to identify root causes of quality problems using data and statistical analysis tools. Through rigorous analysis, you can determine whether the capability of a process meets customer requirements. This establishes the interrelationship between variables and identifies which of the X variables are vital to the Y response. In fact, the Analysis phase provides answers for these questions:

- Does a combination of Xs affect Y?
- If an X input is changed, does it change the output Y?
- How does a change in an X input affect the other Xs?
- And, how much change can X absorb according to other Xs?

What are analyze phase tools?

Several statistical tools are used by the Six Sigma project team during the Analyze phase of the Define, Measure, Analyze, Improve, and Control (DMAIC) methodology.

Tool	**Description**
Multi-vari studies	• A tool used to identify how outputs of a process are affected by inputs
Hypothesis testing	• A formal way to test for differences in process performances regardless of data types and to recognize relationships between samples and populations
Analysis of Variance (ANOVA)	• A hypothesis testing tool used to examine whether the means of various samples differ from one another, assuming that the sample populations are normally distributed
Correlation	• A technique used to identify relationships between two variables

Regression analysis	• A statistical tool used for predicting the outcome of a dependent variable based on the interactions of one or more independent variables

What are analyze phase deliverables?

In the Analyze phase, the root cause or causes (or the vital Xs) of a problem are identified and validated statistically using hypothesis tests, ANOVA, correlation analysis, t-tests, regression and chi-square analysis. During the Analyze phase, it is important to establish the strength of the relationship between Y and Xs. By the end of the phase, you will have identified the improvement objective of Y.

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Perform Multi-Vari Studies

In the previous topic, you described the Analyze phase and the tools and deliverables of this phase. An important tool used during the Analyze phase is a multi-vari chart, which graphically analyzes the stability of a process. In this topic, you will perform multi-vari studies.

A multi-vari chart is a useful graphical tool that shows patterns of variation in a quality characteristic from several possible causes on a single chart or set of charts. The purpose of the chart is to help identify the factor or factors having the greatest effect on variability. Being familiar with different types of variations will help you analyze these issues and plan appropriate remedial actions.

What are the various types of variations that a multi-vari chart can identify?

Essentially, three types of variations are defined in terms of process operations.

Variation Type	**Description**
Cyclical	• Occurs in a sequence among products, or in a process or a series of processes. It is also referred to as intra-piece variation
Positional	• Occurs within a specific product. It is also referred to as inter-piece variation
Temporal	• Occurs in a process over a period of time

Explain cyclical variations with an example.

Cyclical variation is the variation that exists between consecutive outputs of process steps. It represents changes from one process cycle to another process cycle. For example, it can be a variation that occurs due to different lots or batches, or a variation identified from piece to piece or unit to unit.

Cyclical variations are indicated when the same functions performed by different operators or on different machines produce different results. It often describes a repeating pattern and potential causes related to inconsistencies in a process setup. If the cyclical variation is large, then it may be because of the wrong material used, incorrect quantity, at the wrong time, or an unqualified person. It also reveals a longer range periodic pattern of trend variation. Typically, cyclical variation may vary with tool life and will improve every time a new tool replaces a worn one.

Example: Variations in the Surface Finish of a Component
A manufacturer was experiencing variation in the surface finish of a component. On examination, it was found that the surface finish was varying in a cyclical manner. A new tool produced the surface finish well within the specification; however, as the tool aged, the surface finish deteriorated and resulted in components that did not meet the specification. It was observed that when the tool was replaced, the surface finish improved. The team concluded that the variations in the surface finish of a component described cyclical variations in relation to the tool life.

Explain positional variations with an example.

Positional variation is the variation that exists within a process and implies that the variation in the output occurs by virtue of position. In this case, the term position is both literal and figurative.

For example, the variation that exists between different sets of teams or different machines in the same process is also known as positional variation. It also includes variations within a batch of components, in a setup, within the machine position or in a measurement system. Therefore, it is attributable to a process or product design. Positional variation demonstrates clearly defined pattern and is often known as within part variation because it refers to characteristic of the same product.

Example: *Variations in Molded Plastic Parts*
An auto plastic component vendor, a supplier of small components such as door handles, was experiencing high customer rejection. A team was instituted and explicitly tasked with identifying the cause for rejections. Because the mold consisted of eight cavities, they observed the outputs from different cavities of the same mold. The team observed that some cavities had higher rejection rates than other cavities, and this variation in rejection rates that exists between different mold cavities pointed to positional variation. The variations ceased once there commended corrective actions were implemented.

Explain temporal variations with an example.

Temporal variation is the variation that occurs over time. It is also the shift in the process output observed over time. It occurs as change over time and describes the variations in day-today or week-to-week outputs. It is found from setup to setup and from shift to shift for machines or jigs. Season-to-season variations due to temperature or humidity changes are indicative of temporal variations.

Normally, such variations indicate the decay of a process element over time. Patterns of charting data over time that indicate a temporal variation can point to causes by identifying when variations occur.

Example: *Variations in Components Manufactured on Friday and Monday*
The seat assembly plant of an automobile component supplier registered a significant increase in rejections in the batch of components that were manufactured on Fridays and Mondays. An in-depth review of employees' time records showed that these two days accounted for large scale absenteeism. This meant that those present were overloaded with work, and in order to lighten their load, casual workers were employed. Because a majority of the casual workers were inexperienced, this resulted in variation in the output. This variation is a function of time, and resulted in variations in the component.

Describe multi-vari chart with an example?

A *Multi-vari chart* is a tool that graphically depicts variation of a quality characteristic for multiple factors. The purpose of the chart is to permit identification of the factor or factors having the greatest effect on variability. It provides a graphical display of behavior of a Critical to Quality parameter (CTQ) in a running process.

A multi-vari chart aims to assess the extent to which the variability in a quality characteristic can be attributed to the effect of two or more specific factors. Typically, in multi-vari studies, one of these factors is related to time—hours, days, or weeks—and one or more factors are related to streams of products—batches, lots, machines, and operators.

In addition, Y data must be continuous and X data must be categorically discrete—having factors with different levels. It shows the variation within a process, a machine, a part, or even between parts. A multi-vari chart can display piece-to-piece variability, time-to-time variability, and even variability on a single piece and enables the study of process inputs and outputs on a natural day-to-day setting. A multi-vari chart is an excellent tool for quantifying major components of variation—such as temporal, cyclical, and positional—and isolating the dominant type of variation in a process.

Example: Variation in the Weight of Rubber Components

Rubber components molded by a molding machine were consistently showing variation in weight. The quality engineer (QC), charged with analyzing the cause of variation suspected that the variation was either due to the mold cavity or the operator, which were the most common causes of variation. The QC engineer suspected that positional variation could be the cause because the mold had five cavities. Similarly, he focused on analyzing whether the operators were the cause because there were four operators working on the machine. He also considered whether the variation could be due to the interaction of the operators and the mold cavity. Therefore, he collected weights of several samples molded in different cavities and by different operators. A multi-vari chart was used to plot the data and identify the cause of variation.

What are the benefits of multi-vari charts?

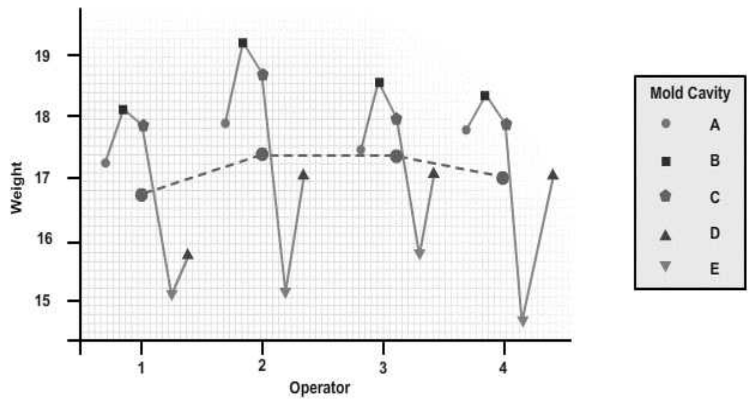
Multi-vari charts are graphical tools that non-statisticians can use and understand. The multi-vari charts can:

- Identify the vital few by reducing the number of Xs
- Graph the influence of up to four factors on a CTQ in a single instance
- Plot data that are either ordinal or nominal in nature
- Plot historical data because the charts are not dependent on a structured data collection plan
- Identify positional, temporal, and cyclical variation in processes
- And, identify the assignable causes of variation

How to interpret of multi-vari charts?

A multi-vari chart is used to interpret the mean at each factor level for every factor. It is also used to interpret the critical set of variation that contains the assignable cause of the variation in a process. The vertical straight line in the chart represents the means of the first factor, which in this instance is the mold cavity at each level of the second factor, which is the operator.

The vertical straight lines in the chart connect the means of the mold cavity within each level of the operator. The horizontal line connects the means of each level of the operator with the vertical line.



A Multi-Vari Chart for Weight by a Mold Cavity-Operator

The multi-vari chart displays the weight results achieved through mold cavity analysis. The vertical straight lines in the chart show a higher degree of variation than the horizontal line.

This indicates that the mold cavity causes a higher variation in weight than the operator. Moreover, the horizontal line is nearly flat, implying that all four operators are nearly consistent when it comes to the weight of the component. The vertical lines for all four operators are nearly the same shape. This implies that there is no interaction between the operator and mold cavity. An analysis of the vertical lines reveals that the mean for cavity D is the lowest and it is the main source of variation in weight among the five cavities. Therefore, the multi-vari chart clearly shows that depending on the desired weight of the component, the respective mold cavities can be corrected.

How to create multi-vari charts in minitab?

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Perform Simple Linear Correlation

In the previous topic, you performed multi-vari studies. In order to accurately analyze a process, a simple linear correlation should also be performed. In this topic, you will perform a simple linear correlation.

While analyzing data, you are exposed to different variables related to various business processes.

Familiarizing yourself with correlation enables you to determine whether two variables are related to one another in terms of their data and the business process.

Explain correlation with an example.

Correlation is a statistical technique that determines the degree of a relationship between two or more variables. When a change in one variable causes a change in the other variable, then there is a correlation between the two variables. Six Sigma project teams use correlation for predicting the effect of one variable on the other in business processes.

Example: Correlation between Productivity of Technicians and Their Experience

A leading automobile equipment manufacturer had a team of qualified technicians who performed necessary quality tests on manufactured goods before they were packaged for shipment. These technicians required technical qualifications, knowledge, and practical experience to perform their jobs. The technicians' productivity was measured in terms of the number of products they tested per day. The data collected on their productivity reveals that technicians with more experience had higher productivity. Therefore, productivity is directly correlated to experience.

What are the types of correlation?

Several types of correlation are available.

Correlation Type	**Description**
Positive	• Between two variables moving in the same direction, when there is an increase in the value of one variable, the value of the other variable also increases
Negative	• Between two variables moving in the opposite direction, when the value of one variable increases, there is a decrease in the value of the other variable
Zero	• A change in the value of one variable does not have any effect on the value of the other variable
Linear	• When there is an increase in the magnitude of one variable, it causes a proportionate change in the magnitude of the other variable

What is correlation coefficient?

The *correlation coefficient,* also known as *Pearson's correlation coefficient,* is the measure of the degree of correlation between two variables. It is denoted by the symbol "r." Typically, a correlation coefficient can range from -1.00 to +1.00, and a value of -1.00 represents a perfect negative correlation. A value of +1.00 represents a perfect positive correlation, while a value of0.00 represents a lack of correlation.

For example, the strength of the correlation between the productivity and experience of technicians can be numerically expressed through the correlation coefficient r = 0.98. The value 0.98 represents a strong positive correlation because as experience increases, productivity also increases. Scatter diagrams are used to graphically depict the correlation coefficient. Usually, scatter diagrams and the correlation coefficient are used simultaneously for decision making.

How to interpret correlation?

Based on the correlation coefficient, you can determine the strength of the correlation between the two variables being examined. If the correlation coefficient is between 0.8 and 1, it is considered a *strong correlation*. On the other hand, if the correlation coefficient is less than 0.6, it is considered a *weak correlation*. If the correlation coefficient is between 0.6 and 0.8, it is regarded as a *moderate correlation*.

What do you mean by statistical significance in correlation?

Statistical significance for a correlation means that the correlation between two variables is unlikely to have occurred by chance or due to an error. When the correlation between two variables is considered statistically significant, it means the differences are likely to be real and probably due to a causative factor. The *p-value* represents the probability that the correlation coefficient is zero, and statistical significance is validated through the p-value. It has a value ranging from 0.00 to 1.00. If p is less than 0.05, it is concluded that the coefficient is statistically significant. Similarly, if p is less than 0.001, it is regarded as statistically highly significant because it means that there is less than a one in a thousand chance of being wrong.

In Six Sigma, statistical significance is applied to establish the hypothesis that there has been a meaningful change in the metric. It is used to confirm the validity of data.

What is causation?

Causation is the cause-and-effect relationship that exists between two variables being studied. It establishes the fact that it is not mere statistical correlation that exists between the variables.

Causation means one variable X causes another variable Y to occur; if a change occurs in X, a resultant change occurs in Y. In causation, X always precedes Y, thereby establishing a time order. Predicting, determining, and controlling the quality of the output becomes easier if the cause-and-effect relationship between output and action is understood. Similarly, understanding causation is the key to performing a regression analysis successfully. It identifies the variables to include in an analysis and helps in understanding why a problem or a defect occurs by providing evidence.

Example: Increase in Power Consumption in a Steel Rolling Mill
The consumption of electrical power in a steel rolling mill considerably increased between October and February. The production manager believed that the increase in electrical power consumption was due to the increase in production volumes. This conclusion was supported by data that established a positive correlation of 0.7. However, the finance manager was not convinced with this explanation because he found that there was an increase in the unit manufacturing cost of steel sheets. He commissioned an external consultant to study and report his findings. The external consultant performed a cause-and-effect study and concluded that there was no causation between the increase in electrical power consumption and production volumes, though a correlation existed. Further, he concluded that the causation was due to a decrease in ambient temperature in these months, and this led to the need for more heating and more power consumption.

What is the difference between correlation and causation?

It is essential to distinguish between correlation and causation because they are not the same.

When two variables are correlated, there may or may not be a causative connection. A correlation is valid only if a cause-and-effect relationship exists, between the variables under investigation. This is validated by process knowledge. Theoretically, it is easy to distinguish between causation and correlation in a process; unfortunately, when it comes to determining them, a lot of confusion exists.

A causation study can be done to determine what would happen if one variable was changed, whereas a correlation analysis can be done to observe the outcome of the two events and offer statistical data as proof. It is extremely difficult to establish causation between two correlated variables.

Correlated occurrences may be due to a common cause. A correlation may also be observed when there is causation behind it, but in order to establish a cause, the possibility of any other explanation for the observed correlation must be ruled out. The most effective way of establishing causation is through a controlled study. One can establish a correlation only if the effects are notable and there is no reasonable explanation that challenges the causation.

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Perform Simple Regression

In the previous topic, you performed simple linear correlation. Another important technique used to analyze a process is regression. In this topic, you will perform simple regression.

During the Analyze phase of the DMAIC methodology, you may come across instances that involve identifying associations between two variables. Your knowledge about regression will help you estimate one variable based on the value of another.

Explain regression with an example.

Regression is a technique that investigates and models the relationship between a dependent variable (Y) and independent predictors (Xs). It is used for hypothesis testing, modeling causal relationships Y = f(x), or predictive modeling. However, it is important to make sure that the underlying model assumptions are not violated. Key outputs in a regression analysis are the regression equation and R2 coefficient. The model parameters are estimated from the data using the method of least squares. The model should also be checked for adequacy fits and residuals.

Example: Quantifying the Relationship of Factors that Impact Car Mileage
A passenger car manufacturer wanted to guide customers on how to improve the mileage of their cars. In order to give them the right directions, the manufacturer decided to first determine the strength of the relationship between the mileages of the passenger car in city driving conditions to the factors that impacted it. A regression example was constructed to quantify the relationship of the factors that impacted the mileage of a passenger car in city driving conditions.

It was found that the tire pressure, density of fuel, number of gear shifts, weight of theca (with passengers), volume of fuel additives added, and average driving speed were strongly correlated with mileage.
The manufacturer developed a regression model using the 140 data points collected.

City mileage	5.4 + 0.015*
Density of fuel	+0.05*
Tire pressure	-0.413*
No. of gear shifts	-0.007*
Weight of the car	+0.093*
Vol. of fuel additive added	+0.095*

Break-up of data points

The asterisk (*) denotes the average driving speed.
Using the above model, the car manufacturer was able to determine the city mileage for different settings of the factors. For example, they were able to determine that if the average driving speed was 10 miles per hour (MPH), the mileage will improve by 0.38 miles per gallon(MPG) if the driver were to average a driving speed of 14 MPH. The passenger car manufacturer extrapolated this finding to guide customers on how they can improve their mileage.

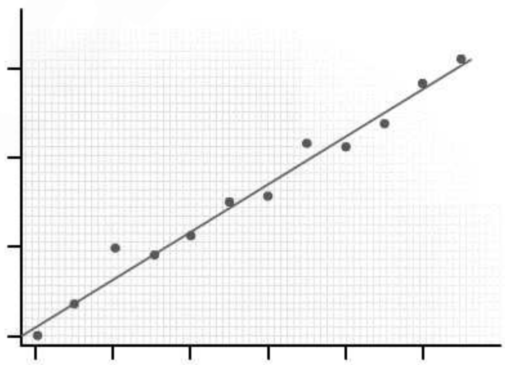
What are the types of regression models?

The most common type of regression is simple linear regression, which assumes a linear relationship between a single X and the Y. Multilinear regression also gives a linear equation but includes several Xs. Nonlinear regression is not commonly used except in instances where the relationship is complex.

Regression Model	**Description**
Simple linear	Assumes a linear relationship between a single X and the Y.
Multilinear	Also assumes a linear equation but includes several Xs.
Nonlinear	Not commonly used except in instances where the relationship is complex.

How is linear regression equation constructed?

The simple linear regression method determines the relationship between a continuous process output (Y) and one continuous factor (X). The relationship is typically expressed in terms of a mathematical equation, such as Y = Y intercept + mX, where Y is the process output, the Y intercept is a constant, m is the slope, and X is the process input or factor. The equation is arrived at by constructing the line of best-fit from the scatter diagram. The simple linear regression equation is also called the least squares regression equation because it is the line for which the sum of squared residuals is at a minimum.



A best-fit line demonstrating the linear regression equation

There are two primary reasons for fitting a linear regression equation to a set of data: to describe the data and to predict or forecast the conditional expected value of an output variable.

If the linear regression equation is used for prediction, or forecasting, it can be used to fit a predictive model to a set of Y and X values. Then, if an additional value of X is given without the accompanying value of Y, the fitted model can be used to predict the conditional expected value of Y. Similarly, if given a variable Y and a number of variables related to it—such as Xi, Xj..., Xp—then a linear regression equation is applied to quantify the relationship between Y and Xj. It is used to determine whether Xj has a relationship with Y at all and to identify the subsets of Xj that have redundant information about Y. It can also be used to infer that other subsets are not informative once one of them is known.

What is R-SQ value?

R squared (R-SQ or R2) is a measure of the validity of the regression equation. It is also called the coefficient of determination. R2 represents the percent variation in Y explained by X.

If the R2 value is greater than 0.65 (65 percent), it represents a significant relationship between X and Y. If the value is less than 0.65, then the regression equation is not complete. In most instances, the factor X alone cannot explain most of the variation in Y and in order to improve Y, you have to either identify other factors which have more significant impact on Y or include additional factors to X. Irrespective of whether the regression equation is linear, nonlinear, or more, a regression analysis yields R2 as an indicator of how well a model fits data.

What is least square method?

The *least square method* is a method to determine the true best fit model that minimizes the total deviation from the model. The least square method places the fitted line at such a place where the error, statistically known as residual, is minimized. It is the line of best fit that represents a correspondence between two measured quantities. The least square method is typically used to determine the best fitting line when the measurements are plotted as points on a graph and fall near the same line. This process is called regression, or linear regression, when the fitted curve is a straight line.

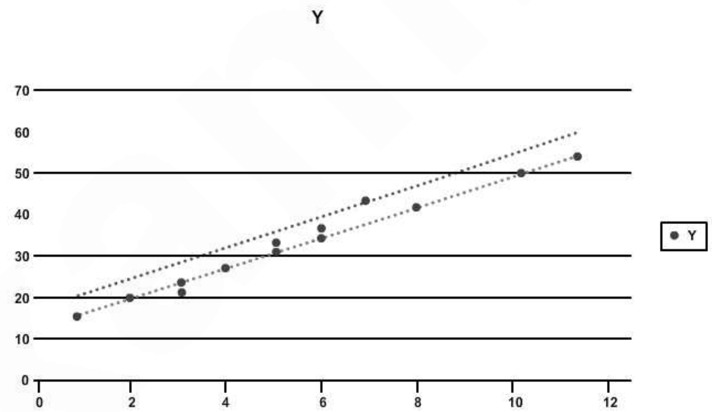


Figure: A best-fit model demonstrating the least square method

How to perform simple regression?

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16 Hypothesis Testing

Introduction to Hypothesis Testing

In the previous lesson, you used multi-vari charts, interpreted results of multi-vari charts, and reviewed and demonstrated concepts of correlation and regression. In addition to these methods, hypothesis testing can be employed to validate the relationship between independent and dependent variables. In this lesson, you will explore key hypothesis testing concepts and steps for testing hypotheses.

The Analyze phase in Six Sigma examines the statistical significance of relationships between process inputs and outputs. Hypothesis testing is an important tool in this process.

It helps validate relationships between these inputs and outputs using objective methods.

Identifying the relationship between process inputs and resultant process outputs in the Analyze phase is to zero in on the variants of the root cause of a problem. This is where hypothesis testing gains significance because it can establish the relationship between causes and effects.

So, it becomes doubly important to match key concepts of hypothesis testing with their descriptions. In this topic, you will describe hypothesis testing.

Defining correct testing parameters is an important factor in the success of a Six Sigma project. If correct test parameters are employed, they enhance performance improvement effort.

The success of a Six Sigma project is jeopardized if incorrect test parameters are employed.

Therefore, it is important to employ hypothesis testing for accuracy.

What is hypothesis testing?

Hypothesis testing is the process of verifying the relationship between the dependent variables, or Ys, and independent variables, or Xs, of a study. It helps analysts accept or reject the tentative relationship between Ys and Xs. Hypothesis testing involves the use of statistical tools to make unbiased and objective decisions. It is a statistical method to validate assumptions about cause-and-effect relationships in Six Sigma projects. However, the effectiveness of hypothesis testing depends on the objectives and parameters defined for the testing.

Example: Productivity of a Stamping Machine
An automobile manufacturer discovered that a sheet metal stamping machine was manufacturing just 30 components per hour. Management formed a team to identify opportunities to improve productivity. The team concluded that reducing the loading and unloading time for components could improve overall productivity. Hypothesis testing was essential to validate the relationship between the loading and unloading time for components and the productivity of the stamping machine.

What are the benefits of hypothesis testing?

Hypothesis testing provides a schematic method to test tentative relationships between variables, regardless of the data type. It is a formal tool that identifies the relationship between samples and populations.

Hypothesis testing helps:

- Identify root cause variations

- Verify how new solutions have improved process performance
- And, avoid validations through trial and error methods

Explain null and alternative hypothesis with an example.

Two hypotheses have to be formulated to statistically validate every assumption: the null hypothesis and the alternative hypothesis. Generally, the null hypothesis is represented withH0, and the alternative hypothesis is represented with Ha.

The *null hypothesis* is a hypothesis which asserts that the independent variable has no influence over the dependent variable. The difference observed between test groups and experimental groups is attributed to chance.

The *alternative hypothesis* is a hypothesis that asserts the existence of a relationship between independent and dependent variables. It also states the nature of the relationship between the variables. In addition, it is the relationship that should be accepted if null hypothesis gets rejected.

Example: Productivity of a Sheet Metal Stamping Machine
An automobile manufacturer aimed to improve the productivity of a sheet metal stamping machine. The team formed to identify the opportunities to improve the productivity of the stamping machine suggested several improvement measures.

The team performed a pilot test to validate the improvement measures. It collected the data of15 samples before and after the implementation of the improvement measures.

Null Hypothesis	Alternative Hypothesis
Null hypothesis was stated as “the hourly output before improvement is the same as the hourly output after improvement.”	Alternative hypothesis stated as “the hourly output after improvement is greater than that of before improvement.”
Ho: Mean of hourly output (before Improvement) = Mean of hourly output (after Improvement)	Ha: Mean of hourly output (before Improvement) <Mean of hourly output (after Improvement)

What is difference between framing null and alternative hypothesis?

Null Hypothesis	Alternative Hypothesis
• Asserts that the independent variable has no influence on the dependent variable. • The difference observed between test groups and	• Asserts the existence of relationship between the independent and dependent variables. • It states the nature of relationship between the variables.

experimental groups is attributed to chance.	
• For example, a Six Sigma project team is comparing process A with process B. If the mean of process A is μ1 and mean of process B is equal to μ2, then the null hypothesis is: $Ho:\mu1 = \mu2$	• Alternative hypothesis can be stated in three different ways, for example: ○ Mean of process A is not equal to mean of process B. $Ha:\mu1 \neq \mu2$ ○ Mean of process A is greater than mean of process B. $Ha:\mu1 > \mu2$ ○ Mean of process A is less than mean of process B. $Ha:\mu1 < \mu2$

In a study, accepting the null hypothesis leads to rejecting the alternative hypothesis, and rejecting the null hypothesis leads to accepting the alternative hypothesis.

What do you understand by statistical significance? Elaborate with an example.

Statistical significance is the probability that the result of a given study could have occurred purely by chance. It reflects the degree to which observed results are true. Hypothesis tests aim to determine if the observed difference is statistically significant.

Example: Improving Hourly Output

An automobile manufacturer sent a team to study the impact of improvement steps on the hourly output of a stamping machine. For this, the team conducted a hypothesis test at the 0.05significance level. From the test results, the team rejected the null hypothesis at the 0.05 significance level. Therefore, the team concluded that the hourly output of the stamping machine increased after improvement.

What do you understand by practical significance? Elaborate with an example.

Practical significance evaluates whether the observed difference is large enough to have any practical impact on a process under study. It evaluates the practical use of a study's outcomes.

A hypothesis test evaluates statistical significance, whereas practical significance evaluates the significance of results considering all practical conditions. It is an inclusive decision for the process owner.

Example: Practical Significance of Improvement

An automobile manufacturer studied the impact of improvement steps on the hourly output of a stamping machine. Statistical studies concluded that there was a significant improvement in the hourly output of the machine after the improvement. The mean before improvement was 30components per hour, and after improvement it increased to 33 components per hour. The manufacturer considers that an improvement of three components per hour is practically insignificant.

Explain the difference between statistical and practical significance.

Statistical significance depends on small population differences and sample sizes, whereas practical significance looks at whether the difference is large enough to be a value in a practical sense.

Sometimes, a hypothesis test can find a claim to be statistically significant. However, a claim may not be worth the effort or expense to implement. Therefore, the organization should always consider practical significance along with statistical significance in a decision-making process. Analysts need to combine engineering judgment with statistical analysis.

What are significance levels?

A significance level is decided by an analyst prior to a hypothesis test. It is the level of error that an analyst is willing to accept while making the decision, and reflects the degree of the test's reliability. It is referred to as the alpha (α) level. Generally, this is set at a 0.05 (5 percent) or 0.01 (1 percent) level.

What do you understand by tests of significance?

A *test of significance* is the process that tests the validity of a claim being made about a population, based on the outcome of studies conducted on a sample. These hypotheses tests use statistical principles to draw such inferences.

In this process, analysts collect sample data and use statistical inference to assess evidence on some claim about the population. This process involves framing null and alternative hypotheses.

The null hypothesis is either rejected or accepted based on statistically significant evidence. It also concludes that observed evidence is not due to a sampling error.

Example: Statistical Significance to Test a Hypothesis
An automobile manufacturer sent a team to study the impact of improvement steps on the hourly output of a stamping machine. The team decided to conduct a hypothesis test at a 0.05significance level. The team set null and alternative hypotheses. Data was collected from 15samples before and after improvement, and the team used statistical methods to test the hypothesis. From the test results, the team rejected the null hypothesis at a 0.05 significance level. Therefore, the team concluded that the hourly output of the stamping machine had increased after improvement.

What are the types of errors?

In the process of testing a hypothesis, analysts can make two types of errors: Type I and Type II.

Error Type	**Description**
Type I error	• An error of rejecting a null hypothesis (Ho) when it is true • Denoted by α and is also called a false positive

	• An error of observing a relationship between variables when there is no relationship between them, therefore known as excessive credulity
Type II error	• This is an error of accepting that a null hypothesis (Ho) is true • Denoted by β and is also known as a false negative • An error of failing to observe the actual relationship between variables, therefore known as excessive skepticism

Illustrate the different outcomes of a hypothesis test based decision.

A hypothesis test can lead to four different outcomes. The table shows the outcomes of any hypothesis testing with Type I and Type II errors.

Result of Test	Population	
	$\mu = \mu_0$	$\mu \neq \mu_0$
Do Not Reject	Correct	Type II Error
Reject	Type I Error	Correct

Figure: *Four different possibilities of a hypothesis test*

Minimizing Type I and Type II errors is a challenging task. Efforts to reduce the probability of one error will increase the probability of another error. Both the errors cannot be reduced simultaneously. Depending on the real-life penalties involved in the decision, decision makers control the probabilities of each error.

Explain power of a test with an example.

The *power of a test* is the probability of rejecting a null hypothesis when it is false. In simple terms, it is the probability of not making a Type II error. If β is the probability of making a Type II error, then it should be small. Therefore, 1 –β should be large for a hypothesis test.

The value 1 –β indicates the power of a test.

The power of a test depends on various factors. Usually it is set at 80 percent or 0.8.

Factor	**Description**
Effect size	• The greater the effect size, the greater the power of the test
Significance level of α	• With an increase in α, β decreases, thereby increasing the power of the test

Sample size	• With an increase in the sample size, the power of the test also increases

Example: Power of Hypothesis Testing
An automobile manufacturer sent a team to study the impact of improvement steps on the hourly output of a stamping machine. For this, the team decided to conduct a hypothesis test at a 0.05 significance level and set null and alternative hypotheses. The team decided on a control Type II error at 0.2, or 20 percent, per the industry standard. Therefore, the power of the testis:

Formula for Power of the Test

$$(1 - \beta) = 1 - 0.2 = 0.8 \text{ or } 80\%$$

Effect Size
Effect size measures the degree of change that the test would reveal.

Explain one-tailed tests.

A *one-tailed test* is a hypothesis test in which the critical, or rejection, region to reject the null hypothesis is located at one end of the distribution. If the test statistic falls in the critical region, the null hypothesis will be rejected and the alternative hypothesis will be accepted.

There are two types of one-tailed tests: left-tailed tests and right-tailed tests.

Explain left- and right-tailed tests.

For instance, a study intended to test whether the mean of a sample is less than a hypothesized value. If μ was the sample mean and μ0 is a hypothesized mean, then:

$$H0: \mu = \mu0 \text{ and } Ha: \mu < \mu0$$

In this scenario, the critical region to reject the null hypothesis lies on the left tail of the distribution. This is a left-tailed test.

Similarly, if a study aimed to test whether the mean of a sample was greater than a hypothesized value, then:

$$H0: \mu = \mu0 \text{ and } Ha: \mu > \mu0$$

In this case, the critical region to reject the null hypothesis lies on the right tail of the distribution. This is a right-tailed test.

Explain two-tailed tests.

Two-tailed tests have two critical regions that are located on both tails of the distribution. The null hypothesis will be rejected if the test statistic falls in either of the critical regions.

For instance, a study aimed to examine whether the sample mean was equal to a hypothesized mean. In this case:

$$H0: \mu = \mu0 \text{ and } Ha: \mu \neq \mu0$$

The null hypothesis will be rejected if μ > μ0 or μ < μ0. Thus, the test has two rejection regions on both tails of the distribution.

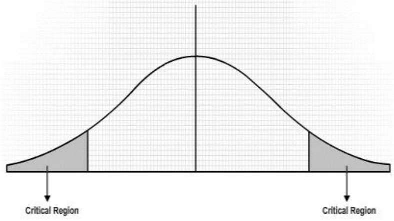

Figure: Critical regions in a two-tailed test

What are critical regions in a two-tailed test?

For instance, a study aimed to examine whether the sample mean was equal to a hypothesized mean. In this case:
H0: μ = μ0 and Ha: μ ≠ μ0

The null hypothesis will be rejected if μ > μ0 or μ < μ0. Thus, the test has two rejection regions on both tails of the distribution.

What graphical plots are used in hypothesis testing?

Graphical methods cannot be used independently to test hypotheses; they must be supported by other statistical testing methods. However, there are two types of graphical plots used for graphical analysis during hypothesis testing.

Graphical Plot Method	**Description**
Box plots	• Used in hypothesis testing than individual plots • Used to assess and compare sample distributions through individual data values
Individual value plot	• Similar to a box plot in identifying outliers and the distribution shape • Plots each value separately

Example:
The research and development (R&D) team of an automobile manufacturer was working on improving the braking efficiency of their motorcycle designs. The braking efficiency is usually measured through stopping distance (in meters) under standard testing conditions. The team decided to perform a hypothesis test to determine whether there was any statistical difference between the braking distances of old and new designs. They collected data for 12 samples for the old design and 12 samples for the new design. They performed a hypothesis test using the box plots graphical method.

From the box plots graph, it is obvious that the mean braking distance of the new design is substantially better than that of the old design. This observation must be checked with standard statistical testing methods.

What factors determine sample size?

The size of a sample plays a critical role in validating the results of a hypothesis test. The sample size should be determined according to the requirements of the study, such as power, effect size, and confidence level.

Factor	Description
Power of the test	• Reflects the test's ability to keep from committing a Type II error. • Increases with an increase in sample size.
Effect size	• Reflects the significance of the relationship between two variables in a population.
Confidence level	• Increases with an increase in sample size.

Example: For a 5 percent confidence interval and a 90 percent confidence level, the sample size should be 272. However, at the same confidence interval and for a 95 percent confidence level, the sample size should be 384.

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Conducting Hypothesis Tests

In the previous topic, you described hypothesis testing and the benefits of hypothesis testing. Furthermore, you identified factors determining sample sizes and appropriate sample sizes.

However, to determine, compare, and contrast statistical and practical significance, you have to perform tests for means, variances, and proportions. In this topic, you will conduct hypothesis tests.

Conducting hypothesis tests is one of the critical tasks of the Analyze phase. It involves multiple stages. Selecting an appropriate hypothesis test is one of them. Meticulously executing a hypothesis test is essential to obtain accurate results that lead to better decisions in later stages.

What are the types of hypothesis tests?

Hypothesis tests can be categorized into three types. They are:

- Hypothesis tests for means
- Hypothesis tests for variances
- And, hypothesis tests for proportions

How to use hypothesis tests for means?

The *hypothesis tests for means* are statistical hypothesis tests that compare means of samples to test a hypothesis. To apply these tests, data should be collected using simple random sampling methods and the population should be normally distributed. If populations are non-normal, the sample size should be greater than 40 or suitable tests have to be used.

Different types of hypothesis tests are used depending on the population size, sample size, variance of the population, and the type of hypothesis test—one-tailed or two-tailed. Generally, student's t-tests are used for hypothesis tests for means.

Example: Testing Efficiency of Brakes

An R&D team for an automobile manufacturer was working on improving the braking efficiency of their motorcycle designs. The braking efficiency is usually measured through stopping distance under standard testing conditions. The team decided to perform a hypothesis test to determine whether there was any statistical difference between the braking distances of the old and new designs. They had to reject the null hypothesis in order to establish that the new design has improved the efficiency of braking.

The team collected data of 12 samples with the old design and 12 samples with the new design. Further, they compared the means of both the samples to check whether the difference between them is statistically different.

How to use hypothesis tests for variances? Give an example.

The *hypothesis tests for variances* are statistical hypothesis tests that use the variances of samples and populations to accept or reject a null hypothesis at a given significance level. To apply this test, the distribution of the population should be normal.

In hypothesis testing for comparing a variance to some hypothesized population variance, chi-square tests are used. It is indicated by X2. If S2 is the variance of a sample and σ^2 is the variance of a population, then:
H0: S2 = σ^2 and chi-square can be calculated by using the formula: where n-1 is the degree of freedom and n is sample size.

The F-test, which is based on F-distribution, is used to test the equality of variances in two normal populations. If σ_1^2 and σ_2^2 are variances of two populations, then:

$$Ho: \sigma_1^2 = \sigma_2^2$$

This hypothesis is tested based on variances of samples drawn from populations. The F-value is the ratio of variance samples.

The higher variance value is always placed in the numerator. Therefore, the F-value is always greater than 1.

Example: Variance in Dispensing Suspension Oil
An automobile suspension manufacturer used an automatic oil dispenser to fill 220 milliliters(ml) ± 5 ml of suspension oil into the fork of a particular model of motor bikes. There were customer complaints regarding the harsh feel of the suspension.

A quality control engineer conducted a root cause analysis and identified that the quantity of oil in the forks was varying beyond the allowed limit of 220 ml ± 5 ml. He decided to replace the filling head of the dispenser with a new one. Furthermore, he decided to use statistical tests to validate if there was a reduction in the variation of the oil quantity.

The engineer collected 15 samples before replacing the dispenser head and 15 samples after replacing the dispenser head. He performed a hypothesis test of variance to determine if the variance has indeed come down. The null hypothesis of the study stated that there was no significant change in the variance after replacing the dispenser head, as compared to the alternative hypothesis that stated that there was a significant difference between variances before and after replacing the dispenser head. In order to prove that the variance had decreased, he had to reject the null hypothesis.

How to use hypothesis tests for proportions? Give an example.

The *hypothesis tests for proportions* are hypothesis tests that are used if samples are drawn using random sampling methods and only two outcomes are possible for each sample point.

This type of hypothesis test uses a null hypothesis and an alternative hypothesis.

The test statistic Z score is used to test the hypothesis. The Z score testing the proportion against a hypothesized proportion is:

$$Z = (p - P) / \sigma$$

Where P is the hypothesized value of population proportion and p is the sample proportion.
Similar to this test, two proportions can also be compared exactly in the same manner. Such a test is called a two proportions test. A null hypothesis is either rejected or accepted by comparing the observed Z score with a critical value for a given significance level.

Example: Decreasing the Proportion of Rejected Shoes
A shoe manufacturing company conducted manual quality checks to test the quality of its finished products. One of the criteria of the quality check was correct pasting of the brand logo on shoes. All the shoes on which the logo is pasted incorrectly will be rejected and cannot be reworked.

The company intended to decrease the proportion of rejected shoes due to incorrect logo pasting.

A team of engineers identified that worn out templates built in the fixtures, which were used for aligning the logos before pasting them, were the cause of errors in logo pasting.
In order to come to this conclusion, the team performed a hypothesis test for proportions. The quality inspector measured the proportion of defective items before and after the replacement of templates for over a period of three days. In order to prove that the proportion of rejection had come down, the team would have to reject the null hypothesis.

What are the criteria for hypothesis test selection?

Hypothesis tests are selected based on the data types of X and Y variables. Criteria for a hypothesis test selection are:

- If X is discrete and Y is continuous, select t-test, paired t-test, or Analysis of Variance
- (ANOVA)
- If X and Y both are continuous, select regression tests
- If X and Y both are discrete, select chi-square tests
- And, if X is continuous and Y is discrete, select logistic regression tests

Explain the process of selecting a hypothesis test.

Selecting a hypothesis test can be a challenging task. The process to arrive at the appropriate test is to first identity the nature of X.

- If X is continuous, select regression tests
- If X is discrete and Y is also discrete, then select chi-square tests
- If X is discrete and Y is continuous and the hypothesis compares multiple groups, then select ANOVA tests
- If X is discrete and Y is continuous and the hypothesis compares two independent groups, then select t-test
- And, if X is discrete and Y is continuous and the hypothesis compares two paired groups, then select paired t-test

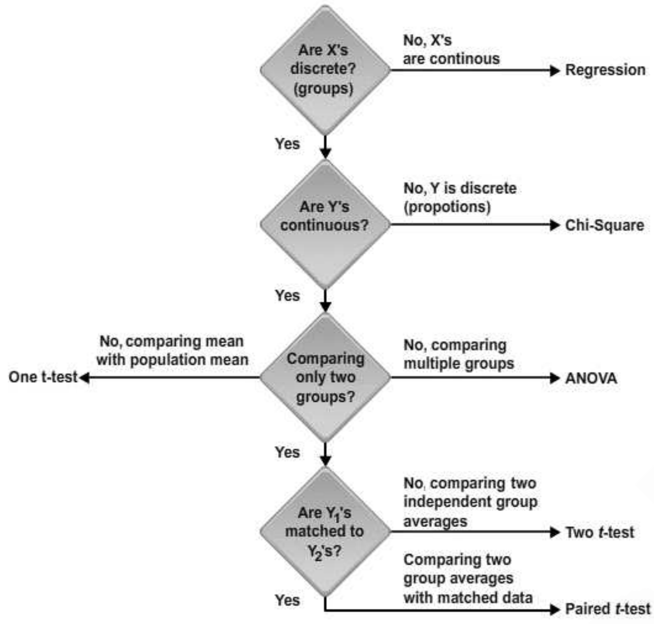

Figure: The flowchart to select a hypothesis test

Explain one sample t-tests with an example.

A *one sample t-test* is a hypothesis test that measures the significant difference between the sample mean and the known or test mean. To apply a one sample t-test, the distribution of Y should be normal or at least free of outliers or skew, the sample must be drawn randomly, and the sample size should be less than 30.

Example: Mean of a Production Process

The mean of a certain production process was 30 with a standard deviation of two. However, the mean of a randomly collected sample of 100 items was found to be 29. In this scenario, the one sample t-test was used to test whether the difference between the mean of the sample and the mean of the population was significant.

Explain two sample t-tests with an example.

A *two sample t-test* is a hypothesis test used to compare two independent population means. The test is based on the assumption that two samples being compared must be independent of each other. If samples are dependent on each other, paired t-test should be used.

The test assumes that values in each sample are independent and normally distributed. Further, the test assumes that two populations have the same variance.

Example: Testing the Increase in the Diameter of a Shaft

A machine shop produced shafts for automobiles. The shop engineer introduced some changes in the machining process to improve the productivity of the shop. The engineer assessed if there was any increase in the diameter of the shaft after the changes in the machining process.

He conducted a two sample t-test using Minitab to compare the shaft diameters before and after the changes in the process.

Two-Sample T-Test and CI: 2t Pre Diameter, 2t Post Diameter

```
N      Mean    StDev   SE Mean
2t Pre Dia   15  30.0046  0.0101   0.0026
2t Post Dia  15  30.0975  0.0128   0.0033


Difference = mu (2t Pre Dia) - mu (2t Post Dia)
Estimate for difference:  -0.09288
95% CI for difference:  (-0.10149, -0.08427)
T-Test of difference = 0 (vs not =): T-Value = -22.09  P-Value = 0.000  DF = 28
Both use Pooled StDev = 0.0115
```

Figure: The Minitab output of a two sample t-test

Explain paired sample t-tests with an example.

A *paired sample t-test* is a hypothesis test used to compare two correlated population means. The term "paired" emphasizes having a correspondence between samples of each population.

Generally, these tests are used to study the effect of a sample manipulation by comparing the sample before the manipulated situation with the sample after the manipulated situation.

The paired sample t-test is based on assumptions that include:

- The populations must be paired
- And, both the populations must be normally distributed

Example: Wear Rate of Brakes

A test engineer in an automobile company aimed to test if the wear rate of brake material had changed after the use of new material B. For this, he conducted a paired sample t-test using Minitab.

Paired T-Test and CI: Pairedt Brake Material A, Pairedt Brake Material B

```
Paired T for Pairedt Brake Material A - Pairedt Brake Material B

                              N     Mean     StDev   SE Mean
Pairedt Brake Material A     15  4.00014  0.01104  0.00285
Pairedt Brake Material B     15  3.89516  0.01069  0.00276
Difference                   15  0.10498  0.01310  0.00338


95% CI for mean difference: (0.09773, 0.11223)
T-Test of mean difference = 0 (vs not = 0): T-Value = 31.04  P-Value = 0.000
```

Figure: The Minitab output of a paired t-test

Explain ANOVA.

Analysis of Variance (ANOVA) is a hypothesis test used to evaluate a hypothesis that involves comparing the means of two or more groups. It generalizes a t-test to more than two groups.

ANOVA is better than a t-test to compare the means of more than two groups because it avoids Type I errors. ANOVA simultaneously tests several sample means to know whether the difference between the means is significant. If the difference is found to be significant, a t-testis used to examine the different pairs to identify where the difference lies.

ANOVA assumes that:

- Each population has the same variance

- Each population is normally distributed
- And, each observation is independent of another

Explain variance with an example.

Variance is the square of standard deviation. Variance of different samples can be added or it can be divided into different components. Therefore, ANOVA can be used to compare variances “within groups” and “between groups.”

Example: Comparing Lengths of Pins

A test engineer of an engineering company aims to ascertain whether the lengths of pins from samples inspected from three different lots are different. He conducts one-way ANOVA to compare three lots using Minitab.

One-way ANOVA: ANOVA Lot 1, Lot 2, Lot 3

```
Source  DF       SS       MS      F      P
Factor   2  1.08709  0.54354  63.63  0.000
Error   33  0.28191  0.00854
Total   35  1.36900

S = 0.09243   R-Sq = 79.41%   R-Sq(adj) = 78.16%


                                  Individual 95% CIs For Mean Based on
                                  Pooled StDev
Level         N    Mean   StDev   -+---------+---------+---------+--------
ANOVA Lot 1  12  8.8932  0.1006   (---*--)
Lot 2        12  8.9788  0.0864         (---*--)
Lot 3        12  9.2971  0.0897                               (---*--)
                                  -+---------+---------+---------+--------
                                  8.85      9.00      9.15      9.30

Pooled StDev = 0.0924
```

Figure: The Minitab output of one-way ANOVA test

Explain the chi-square test. Give an example.

Chi-square is a statistical test used to check a hypothesis by comparing observed data with expected data. The chi-square test is applied if:

- Data is attributed or discrete
- All observations are independent of each other
- The sample size is adequate, preferably more than 10
- Data is collected using random sampling methods
- And, data is arranged in the form of a frequency table

To find the chi-square value, determine expected frequencies from the data frequencies or a frequency set based on prior knowledge. Expected cell counts should not be less than five.

Chi-square is calculated by using the formula:

$$\chi^2 = \Sigma \frac{(O - E)^2}{E}$$

Where O is the observed frequency and E is the expected frequency.

Example: Comparing Machine Defects